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NONSTATIONARY MOTION OF A SHELL ON THE SURFACE OF A HEAVY FLUID

Yu. F. Orlov and A. S. Suvorov

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A three-dimensional nonstationary problem of vibrations of a flexible shell moving on the surface of an ideal heavy fluid. The forces due to surface tension are ignored. The problem is formulated in the space of the acceleration potential. The potential of the pulsating source is found by solving the Euler equation and the continuity equation taking into account the free-surface conditions (linear theory of small waves) and the conditions at infinity. The density distribution function of the dipole layer is determined from the boundary conditions on the surface of the shell. Formulas for determining the shape of gravity waves on the fluid surface and the natural frequencies of vibrations of the shell are obtained.

Key words: *hydroelasticity, gravity waves, potential flow.*

At present, the hulls of some high-velocity vehicles (so-called dynamic air-cushion vehicles [1]) are partly made of elastic materials, allowing an increase in the performance of such vehicles moving in the take-off mode on a water surface. Since in some modes of motion with acceleration or deceleration there may be synchronism between the elastic waves traveling along the ship hull and the gravity waves on the fluid surface, which can result in both a sudden increase in the vibration level, and accidents due to loss of stability of motion, it is necessary to analyze the dynamic characteristics of hulls of this type.

In the present paper, we consider a hull model similar in shape to the hull of the Volga-2 and Raketa-2 dynamic air-cushion passenger ships [1]. It is assumed that the hull has a shallow draft and is a stretched weightless shell of length $2L$ and width $2B$ made of a material which obeys linear Hook law. It is assumed that the fluid on which the hull moves is an ideal incompressible heavy medium of density ρ and on the fluid surface there are no forces due to surface tension. The plane XY of the Cartesian coordinates XYZ coincides with the unperturbed fluid surface. The shell moving in the increasing X direction at velocity U vibrates in such a manner that the velocity of vibration of its points is parallel to the Z axis. The vibration frequency is ω. The acceleration vector due to gravity $\boldsymbol{g}$ is directed along the Z axis. Assuming that the motion of the fluid is potential in space and harmonic in time, the acceleration potential Θ can be written as

$$\Theta = \rho U^2 \, \mathrm{e}^{i\omega t} \, \theta,$$

where t is time and θ is a time-independent function.

Ignoring the nonlinear terms in the equations of motion of an ideal fluid, the boundary-value problem can be formulated in terms of acceleration potential theory as follows [2, 3]:

$$\begin{gathered}
\theta_{xx} + \theta_{yy} + \theta_{zz} = 0, \\
\frac{\partial}{\partial z} \int\limits_{\infty}^{x} \mathrm{e}^{ik(x-\tau)} \, \theta(\tau) \, d\tau \Big|_{z=0} = -\varepsilon_x + ik(\varepsilon + h) \qquad \text{on} \quad S, \\
\theta_{xx} - 2ik\theta_x - k^2\theta + \mu(\theta_x + ik\theta) + \frac{1}{\mathrm{Fr}^2} \, \theta_z \Big|_{z=0} = 0 \qquad \text{outside of} \quad S, \\
\lim_{z \to -\infty} \nabla\theta = 0.
\end{gathered} \tag{1}$$

Nizhny Novgorod State Technical University, Nizhny Novgorod 603600; yorlov@mts-nn.ru; suvorov-a@ya.ru.
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Here $x = X/L$, $y = Y/L$, and $z = Z/L$ are the dimensionless coordinates, $k = \omega L/U$ is the Strouhal number, $\varepsilon(x, y)$ is the hull draft ($\varepsilon \ll L$), h is a constant equal to the wave height at the leading edge of the hull, S is the surface of contact of the hull and the fluid, μ is the dissipation coefficient, and $\mathrm{Fr} = U/(gL)^{1/2}$ is the Froude number.

In the problem of natural vibration frequencies, the shape of the vibrating surface of the hull is an initially unknown function, and, hence, to close the boundary-value problem, we must supplement Eq. (1) with the dynamic elasticity condition in the following form [4]:

$$P_{\mathrm{ext}} = R(\varepsilon_{xx} + \varepsilon_{yy}) - \mathrm{Fr}^2 P_d, \qquad R = N/(\rho g L^2). \tag{2}$$

Here P_d is the dimensionless pressure on the contact surface between the hull and the fluid, R is the dimensionless rigidity of the shell, N is the tension of the shell, and P_{ext} is the external pressure responsible for the vibrations. It should be noted that, generally, the distribution of the external pressure P_{ext} determining the vibration amplitude ε is an unknown function. Therefore, it is reasonable to obtain the solution of the problem for $P_{\mathrm{ext}} \to 0$, i.e., to find the critical parameters of the motion Fr and k for which there is a resonance. This can be done by expanding the function ε in a series of functions which satisfy the boundary conditions in the absence of motion on a certain line l (in the present paper, to simplify the transformations, it is assumed that the line l coincides with the three-phase line in the unperturbed state). In this case, since the ship hulls considered are extended along the x axis, as a first approximation, we can confine ourselves to the first symmetric and asymmetric modes of hull bending

$$\varepsilon_{\mathrm{s}} = -(1 - x^2 - y^2/\lambda^2), \qquad \varepsilon_{\mathrm{a}} = -x(1 - x^2 - y^2/\lambda^2) \qquad (\lambda = B/L) \tag{3}$$

assuming that the resonant modes caused by more complex vibrations are more localized and their effect on the dynamics of translational motion can be ignored. Then, integration of expression (2) yields a system of equations for the external vertical force and the external moment of the different. Assuming that the shell performs free vibrations and the external pressure P_{ext} does not act on it, we obtain the following system of homogeneous equations:

$$\int\limits_S (R(\varepsilon_{\mathrm{c}xx} + \varepsilon_{\mathrm{c}yy}) - \mathrm{Fr}^2 P_{\mathrm{s}})\, dS = 0, \qquad \int\limits_S x(R(\varepsilon_{\mathrm{a}xx} + \varepsilon_{\mathrm{a}yy}) - \mathrm{Fr}^2 P_{\mathrm{a}})\, dS = 0 \tag{4}$$

(P_{s} and P_{a} are the fluid pressures on the hull for the symmetric and asymmetric bending, respectively). The minimum value of the determinant of the system for various forms of the parameters Fr and k corresponds to the modes of motion with increased vibration of the hull.

Thus, to solve the problem, it is necessary to determine the hydrodynamic pressure acting on the ship hull during its bending according to (3) by solving problem (1). The solution of problem (1) can be sought in the form of an integral operator of the type of the double-layer potential [2, 3, 5]:

$$\theta = \int\limits_S P(\xi, \eta) \frac{\partial}{\partial \zeta} G(x, y, z, \xi, \eta, \zeta) dS. \tag{5}$$

Here P is the pressure jump on the surface S, ξ, η, and ζ are the coordinates of the moving pulsating sources that replace the surface of the plate, and G is a Green function. The Green function that satisfies the boundary conditions on the fluid surface and at infinity has the following form [2, 3, 6]:

$$G = \frac{1}{4\pi}\Big(\frac{1}{r_1} - \frac{1}{r_2}\Big) - \frac{1}{2\pi^2}\Big(\text{v.p.} \int\limits_0^\infty \rho\, \mathrm{e}^{(z+\zeta)\rho} \int\limits_{-\pi}^{\pi} \frac{\mathrm{e}^{-i\rho\cos(\varphi)(x-\xi)} \cos(\rho y \sin(\varphi))}{\mathrm{Fr}^2(\rho\cos(\varphi) + k)^2 - \rho}\, d\varphi\, d\rho\Big)$$

$$- \frac{i}{4\pi\,\mathrm{Fr}} \int\limits_{\alpha_1}^\infty \frac{\mathrm{e}^{(z+\zeta)\rho}}{\sqrt{\rho}} \frac{\mathrm{e}^{-i\rho A_1(x-\xi)} \cos(\sqrt{1 - A_1^2}\,(y - \eta))}{\sqrt{1 - A_1^2}}\, d\rho + \frac{i}{4\pi\,\mathrm{Fr}} \int\limits_{\alpha_2}^\infty \frac{\mathrm{e}^{(z+\zeta)\rho}}{\sqrt{\rho}} \frac{\mathrm{e}^{-i\rho A_2(x-\xi)} \cos(\sqrt{1 - A_2^2}\,(y - \eta))}{\sqrt{1 - A_2^2}}\, d\rho, \tag{6}$$

where

$$r_1 = \sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}, \qquad r_2 = \sqrt{(x - \xi)^2 + (y - \eta)^2 + (z + \zeta)^2}.$$

For the integration of the integrand function over φ, the position of the singular points A_1 and A_2 of this function is given by the relation

$$A_{1,2} = -\frac{k}{\rho} \pm \frac{1}{\mathrm{Fr}\sqrt{\rho}},$$

and the integration limits α_1 and α_2 are found from the following equations:

$$\frac{k}{\alpha_1} - \frac{1}{\mathrm{Fr}\sqrt{\alpha_1}} = 1, \qquad \frac{k}{\alpha_2} + \frac{1}{\mathrm{Fr}\sqrt{\alpha_2}} = 1.$$

In view of (5) and (6), the expression for the acceleration potential becomes

$$\theta = \frac{1}{4\pi}\int\limits_S P\frac{\partial}{\partial\zeta}\Big(\frac{1}{r_1} - \frac{1}{r_2}\Big)\,dS - \frac{1}{2\pi^2}\Big(\text{v.p.}\int\limits_S P\int\limits_0^\infty \rho^2\,\mathrm{e}^{(z+\zeta)\rho}\int\limits_{-\pi}^{\pi}\frac{\mathrm{e}^{-i\rho\cos(\varphi)(x-\xi)}\cos(\rho y\sin(\varphi))}{\mathrm{Fr}^2(\rho\cos(\varphi)+k)^2-\rho}\,d\varphi\,d\rho\,dS\Big)$$

$$-\frac{i}{4\pi\,\mathrm{Fr}}\int\limits_S P\int\limits_{\alpha_1}^{\infty}\sqrt{\rho}\,\frac{\mathrm{e}^{(z+\zeta)\rho-i\rho A_1(x-\xi)}\cos\big(\sqrt{1-A_1^2}\,(y-\eta)\big)}{\sqrt{1-A_1^2}}\,d\rho\,dS$$

$$+\frac{i}{4\pi\,\mathrm{Fr}}\int\limits_S P\int\limits_{\alpha_2}^{\infty}\sqrt{\rho}\,\frac{\mathrm{e}^{(z+\zeta)\rho-i\rho A_2(x-\xi)}\cos\big(\sqrt{1-A_2^2}\,(y-\eta)\big)}{\sqrt{1-A_2^2}}\,d\rho\,dS. \tag{7}$$

According to (1), the wave height on the fluid surface is equal to

$$Z_{\mathrm{w}} = -\frac{\mathrm{Fr}^2}{2\pi^2}\Big(\text{v.p.}\int\limits_S P\int\limits_0^\infty \rho^2\,\mathrm{e}^{\zeta\rho}\int\limits_{-\pi}^{\pi}\frac{\mathrm{e}^{-i\rho\cos(\varphi)(x-\xi)}\cos(\rho y\sin(\varphi))}{\mathrm{Fr}^2(\rho\cos(\varphi)+k)^2-\rho}\,d\varphi\,d\rho\,dS\Big)$$

$$-\frac{i\,\mathrm{Fr}}{4\pi}\int\limits_S P\int\limits_{\alpha_1}^{\infty}\sqrt{\rho}\,\frac{\mathrm{e}^{\zeta\rho-i\rho A_1(x-\xi)}\cos\big(\sqrt{1-A_1^2}\,(y-\eta)\big)}{\sqrt{1-A_1^2}}\,d\rho\,dS$$

$$+\frac{i\,\mathrm{Fr}}{4\pi}\int\limits_S P\int\limits_{\alpha_2}^{\infty}\sqrt{\rho}\,\frac{\mathrm{e}^{\zeta\rho-i\rho A_2(x-\xi)}\cos\big(\sqrt{1-A_2^2}\,(y-\eta)\big)}{\sqrt{1-A_2^2}}\,d\rho\,dS. \tag{8}$$

By virtue of the assumptions of shallow draft of the hull, $\zeta \to -0$ in expression (8), i.e., the dipoles are directly under the fluid free surface.

Thus, to calculate the gravity wave distribution, it is necessary to know the pressure distribution on the surface of the shell. For this, problem (1) should be reduced to a two-dimensional integral equation for the variables x and y using the boundary kinematic condition on the surface S. In view of (1) and (7), this equation is written as

$$\lim_{\zeta\to-0}\int\limits_S\frac{P}{8\pi^2}\int\limits_0^\infty\int\limits_{-\pi}^{\pi}\frac{\rho^2\,\mathrm{e}^{-i\rho(\cos(\varphi)(x-\xi)-\sin(\varphi)(y-n))}}{i(\rho\cos(\varphi)+k-i\mu)}$$

$$\times\Big(1+\frac{\mathrm{Fr}^2(\rho\cos(\varphi)+k-i\mu)^2+\rho}{\mathrm{Fr}^2(\rho\cos(\varphi)+k-i\mu)^2-\rho}\,\mathrm{e}^{2\zeta\rho}\Big)\,d\varphi\,d\rho\,dS = -\varepsilon_x + ik(\varepsilon+h). \tag{9}$$

The solution of Eq. (9) is obtained for $\mu \to 0$.

Equation (9) differs from the same equation in the problem of nonstationary motion of an underwater wing [2, 3] in that it does not contain a Cauchy kernel in explicit form. This kernel is found in the limiting cases of motion $\mathrm{Fr} \to \infty$ and $\mathrm{Fr} \to 0$. For arbitrary velocities, the support of the kernel Cauchy are the residues in (7) [5]. Furthermore, integration over the wing span gives rise to one more pole in the expression for the principal value of the integral in (7) because of the occurrence of $\sin\varphi$ in the denominator, and, hence, it is necessary to additionally distinguish a third residue [2, 5]. Panchenkov [3] proposed an effective method for the solution of nonstationary problems — the method of separation of solutions. In the present paper, an approximate solution of the integral equation is constructed by converting to a Fourier transformation of the wave height distribution, which leads to a separation of solutions similar to [3]. This eliminates the difficulties associated with the calculation of the wave integrals in (7).

The wave height defined by expression (8) has the following Fourier transform:

$$\tilde{Z}_{\rm w} = -\frac{\mathrm{Fr}^2\,\tilde{P}\sqrt{m^2+n^2}\,\mathrm{e}^{\zeta\sqrt{m^2+n^2}}}{\mathrm{Fr}^2(m+k-i\mu)^2-\sqrt{m^2+n^2}}.$$

Separating this function in Eq. (9), we write Fourier transform as

$$\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\frac{\sqrt{m^2+n^2}\,\mathrm{e}^{-imx-iny}}{8\pi^2 i(m+k-i\mu)}\Big(\tilde{P}-\frac{\tilde{Z}_{\rm w}\,\mathrm{e}^{\zeta\sqrt{m^2+n^2}}}{\mathrm{Fr}^2}\Big)\,dm\,dn$$

$$= -\varepsilon_x + ik(\varepsilon+h) - \frac{i}{8\pi^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\tilde{Z}_{\rm w}(m+k-i\mu)\,\mathrm{e}^{\zeta\sqrt{m^2+n^2}-imx-iny}\,dm\,dn. \qquad (10)$$

In the region S, the fluid surface coincides with the surface of the shell, i.e., the equality $Z_{\rm w} = \varepsilon + h$ is satisfied. This allows the equation to be to simplified by using the approximation

$$\tilde{Z}_{\rm w}\,\mathrm{e}^{\zeta\sqrt{m^2+n^2}} = C(\tilde{\varepsilon}+\tilde{h}).$$

The constant C, whose value is close to Unity, can be chosen using the least-squares method, minimizing the error of satisfaction of the boundary condition on the surface S, i.e., equality (9). Thus, the wave component of the kernel of the integral equation (9) is taken into account. This approach allows one, by introducing a new function

$$\Gamma = P - C\varepsilon/\,\mathrm{Fr}^2, \qquad \Gamma \in C_0^0(y)\times C_0^\infty(x), \qquad (11)$$

to transform from Eq. (10) to the well-known equations of nonstationary motion of a wing:

$$\frac{1}{2\pi}\int\limits_{\infty}^{x}\mathrm{e}^{ik(x-\tau)}\int\limits_{S}\frac{\Gamma}{r^3}\,dS\,d\tau = (-\varepsilon_x + ik(\varepsilon+h))(2-C). \qquad (12)$$

Here $r = \sqrt{(x-\xi)^2+(y-\eta)^2}$.

Equation (12) can be decomposed into two independent equations:

$$\frac{1}{2\pi}\int\limits_{\infty}^{x}\mathrm{e}^{ik(x-\tau)}\int\limits_{S}\frac{\Gamma_0}{r^3}\,dS\,d\tau = 2(-\varepsilon_x + ik\varepsilon)\Big(1-\frac{C}{2}\Big),$$

$$\frac{1}{2\pi}\int\limits_{\infty}^{x}\mathrm{e}^{ik(x-\tau)}\int\limits_{S}\frac{\Gamma_1}{r^3}\,dS\,d\tau = 2ikh\Big(1-\frac{C}{2}\Big). \qquad (13)$$

Here Γ_0 is the load due to the shell bending and Γ_1 is the loading due to the momentless motion of the shell. In Eqs. (13), the external integral can be eliminated by differentiating them with respect to the coordinate and time, i.e., by transforming from velocities to accelerations on the right sides of the equations. However, this approach leads to a partial loss of the solution during differentiation. Therefore, it is necessary to take into account the load caused by an instantaneous change in the fluid velocity at the leading edge of the shell and to represent the loads Γ_0 and Γ_1 as the sums

$$\Gamma_0 = \Gamma_{\rm dyn} + c_0\Gamma_{\rm imp}, \qquad \Gamma_1 = \Gamma_h + c_h\Gamma_{\rm imp}.$$

Here $\Gamma_{\rm dyn}$ and Γ_h are the components of the total load caused by velocity changes in the region S:

$$\frac{1}{2\pi}\int\limits_{S}\frac{\Gamma_{\rm dyn}}{r^3}\,dS = -(\varepsilon_{xx} - 2ik\varepsilon_x - k^2\varepsilon)(2-C), \qquad \frac{1}{2\pi}\int\limits_{S}\frac{\Gamma_h}{r^3}\,dS = k^2(2-C). \qquad (14)$$

The left sides of Eqs. (14) contain divergent improper integrals only if $\Gamma_{\rm dyn} \in C_0^0(y) \times C_0^0(x)$, $\Gamma_h \in C_0^0(y) \times C_0^0(x)$. The impact component of the load $\Gamma_{\rm imp}$ due to instantaneous velocity changes at the leading edge of the region S (an analog of the singular solution in [3]) is given by the equation

$$\frac{1}{2\pi} \int\limits_{\infty}^{x} \int\limits_{S} \frac{\Gamma_{\rm imp}}{r^3}\, dS\, d\tau = 1, \qquad \Gamma_{\rm imp} \in C_0^0(y) \times C_0^\infty(x). \tag{15}$$

In Eqs. (14) and (15), the kernels do not contain the Strouhal number and the right side are the contributions of the vibrational and translational components of motion of the plate. This allows us to obtain solutions of the equation separately for the vibrational and translational components. Combining these components, it is possible to construct solutions for any Strouhal number that differ only in the values of the constants c_0, c_h, and h. Solutions of Eqs. (15) can be sought in the following form [3]:

$$\Gamma_{\rm dyn} = (H_1 + H_2 x + H_3 x^2 + \ldots)\sqrt{1 - x^2 - y^2/\lambda^2}, \qquad \Gamma_h = D_1(\lambda)\sqrt{1 - x^2 - y^2/\lambda^2},$$

$$\Gamma_{\rm imp} = \begin{cases} D_2(\lambda)\sqrt{1 - x^2 - y^2/\lambda^2} + D_3(\lambda) x^2/\sqrt{1 - x^2 - y^2/\lambda^2}, & x > 0, \\ D_2(\lambda)\sqrt{1 - x^2 - y^2/\lambda^2}, & x \leqslant 0. \end{cases} \tag{16}$$

For $\lambda \leqslant 1/3$, the constant D_2 can be ignored. The values of the constants $H_1, H_2, H_3, \ldots$ depend on the particular bending mode ε. The constants c_0, c_h, and h should be determined as unknown constants of integration in transforming from accelerations to velocities and displacements. For example, the constant h is calculated by the formula

$$h = -\lim_{\mu \to 0} \Big[\Big(\frac{1}{2\pi^2} \int\limits_0^\infty \int\limits_0^\pi \frac{(\tilde\Gamma_{\rm d0} - c_0 \tilde\Gamma_{\rm imp}) r^2}{(r\cos\varphi + k - i\mu)^2}\, d\varphi\, dr - (2 - C)\varepsilon\Big|_{x=0}\Big) \Big/ \Big(\frac{1}{2\pi^2} \int\limits_0^\infty \int\limits_0^\pi \frac{(\tilde\Gamma_h - c_h \tilde\Gamma_{\rm imp}) r^2}{(r\cos\varphi + k - i\mu)^2}\, d\varphi\, dr - (2 - C)\Big)\Big].$$

The total fluid pressure on the shell is defined according to (11):

$$P = \Gamma_{\rm dyn} + h\Gamma_h + (c_0 + hc_h)\Gamma_{\rm imp} + C\varepsilon/\,{\rm Fr}^2\,. \tag{17}$$

For the parameter $\lambda = 1/3$, the values of the unknown constants in expressions (16) are defined for symmetric and asymmetric bending according to expression (3). The load on the shell is defined as

$$(\Gamma_{\rm dyn})_{\rm s} = \frac{2}{3\pi}\Big(k^2\Big(\frac{16}{15} - x^2\Big) - 6ikx + 3\Big)\sqrt{1 - x^2 - \frac{y^2}{\lambda^2}}\,(2 - C),$$

$$(\Gamma_{\rm dyn})_{\rm a} = \frac{2}{3\pi}\Big(\frac{k^2}{2}\, x\Big(\frac{18}{15} - x^2\Big) + i\,\frac{k}{3}\,(10x^2 - 1) + 6x\Big)\sqrt{1 - x^2 - \frac{y^2}{\lambda^2}}, \tag{18}$$

$$\Gamma_h = \frac{k^2}{\pi}\sqrt{1 - x^2 - \frac{y^2}{\lambda^2}}\,(2 - C), \qquad \Gamma_{\rm imp} = \begin{cases} \dfrac{2x^2}{\pi\sqrt{1 - x^2 - y^2/\lambda^2}}\,(2 - C), & x > 0, \\ 0, & x \leqslant 0. \end{cases}$$

Here $(\Gamma_{\rm dyn})_{\rm s}$ and $(\Gamma_{\rm dyn})_{\rm a}$ are the dynamic loads for the symmetric bending ($\varepsilon_{\rm s}$) and asymmetric bending ($\varepsilon_{\rm a}$), respectively. Using formulas (8), (17), and (18), it is possible to determine the shape of the fluid free surface for various modes of motion of the shell. Figure 1 gives the results of calculations of the gravity waves caused by symmetric vibrations of the shell on the fluid surface. Figure 2 shows the calculation results for the waves caused by asymmetric vibrations of the shell (T is the vibration period).

Integration of expression (4) taking into account (17) and (18) reduces the problem to the two homogeneous equations

$$0 = \varepsilon_0\Big[\pi\Big(R + \frac{R}{\lambda^2} + \frac{1}{4}\Big) - {\rm Fr}^2\,\Big(k^2\,\frac{26}{135} + \frac{2}{3}\Big) + \frac{{\rm Fr}^2}{3}\,\Big(c_0 + (k^2 - c_h)h_0\Big)\Big] + \varepsilon_1\Big(\frac{{\rm Fr}^2}{3}\,\Big(c_1 + (k^2 - c_h)h_1\Big) - {\rm Fr}^2\,\frac{2i}{27}\,k\Big),$$

$$0 = \varepsilon_0\Big({\rm Fr}^2\,ik\,\frac{4}{15} + \frac{{\rm Fr}^2}{4}\,(c_0 - c_h h_0)\Big) + \varepsilon_1\Big[\frac{\pi}{4}\Big(3R + \frac{R}{\lambda^2} + \frac{1}{6}\Big) - {\rm Fr}^2\,\Big(k^2\,\frac{3}{175} + \frac{4}{15}\Big) + \frac{{\rm Fr}^2}{4}\,(c_1 - c_h h_1)\Big].$$

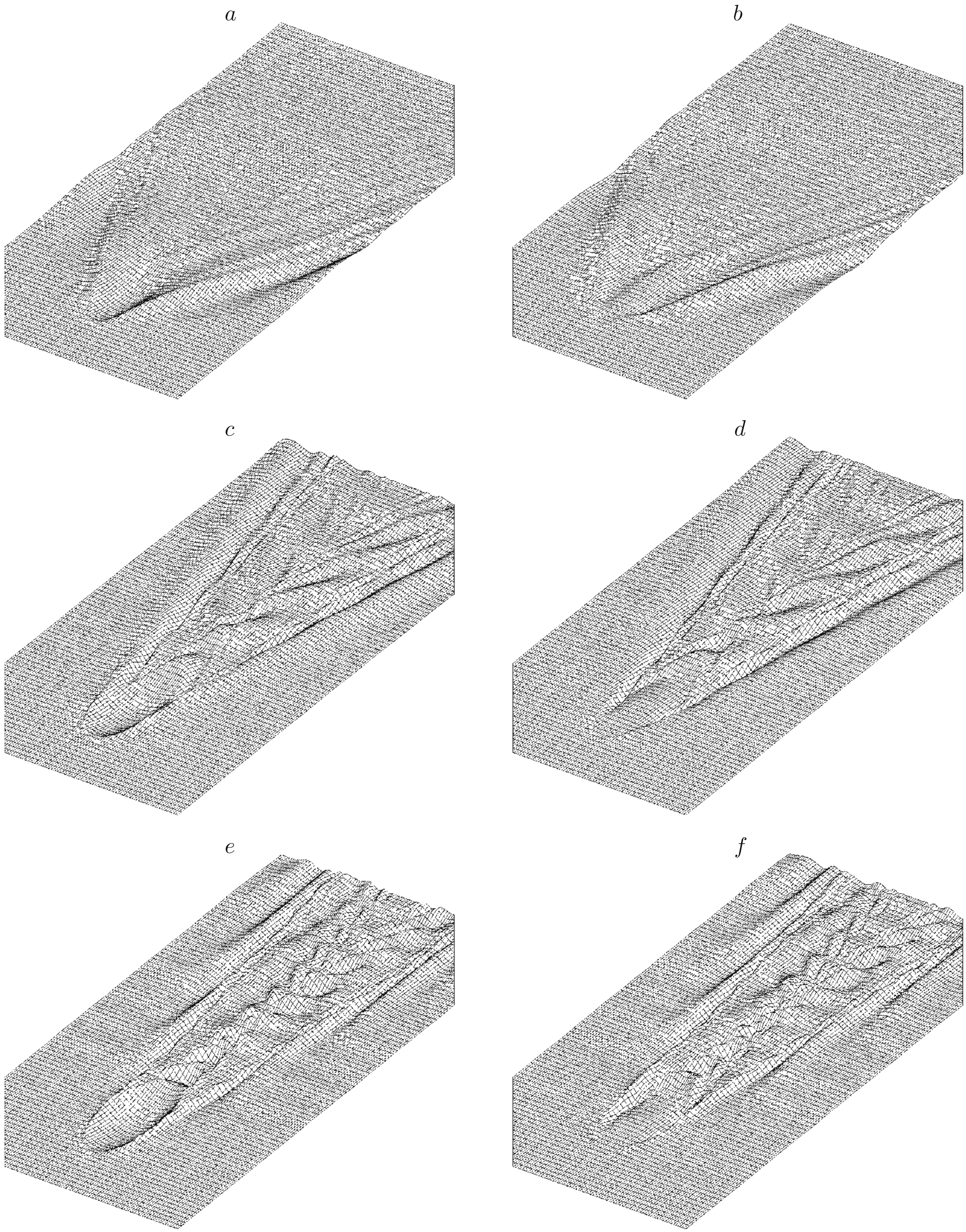

Fig. 1. Shapes of the fluid free surface at the times $t = T$ (a, c, e) and $t = T/2$ (b, d, f) in the case of symmetric vibrations of the shell: $\mathrm{Fr} = 0.3$ and $k = 6.7$ (a and b), $\mathrm{Fr} = 0.8$ and $k = 2$ (c and d), and $\mathrm{Fr} = 0.8$ and $k = 5$ (e and f).

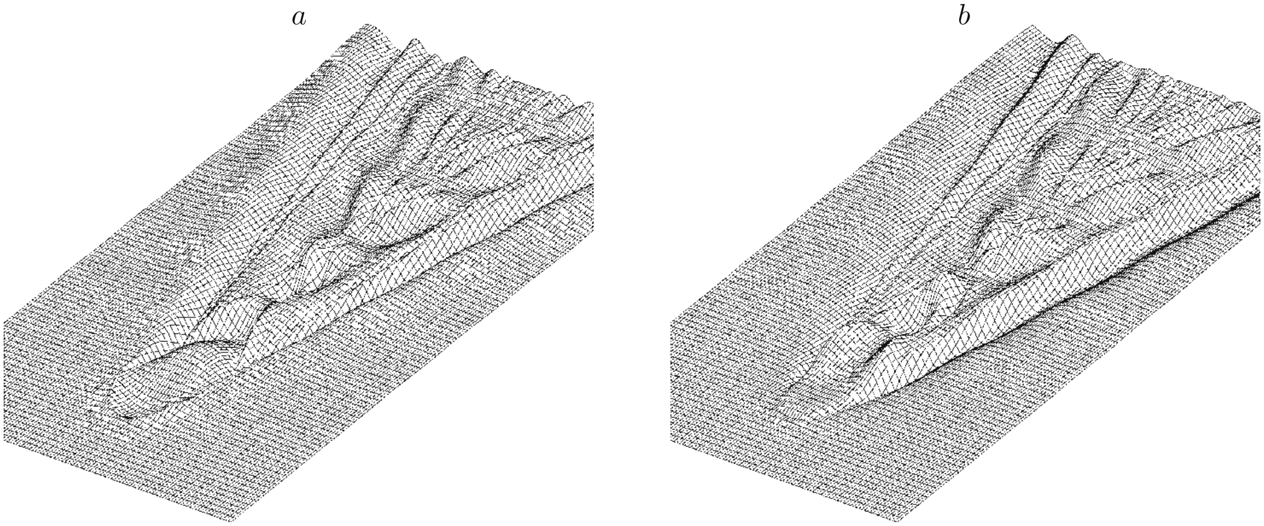

Fig. 2. Shapes of the fluid free surface (Fr = 0.8 and $k = 2$) at the times $t = T$ (a) and $t = T/2$ (b) for asymmetric vibrations of the shell.

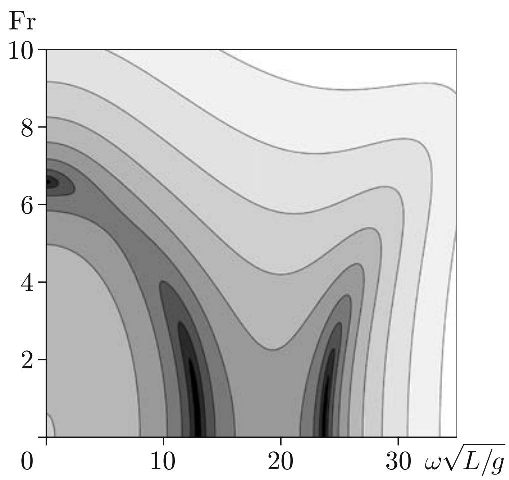

Fig. 3. Vibration amplitude versus vibration frequency and Froude number: the dark regions are the regions in which the values of the determinant are minimal.

The values of the parameters Fr and k for which the determinant of this system of equations is minimal correspond to the resonant mode of motion.

Figure 3 shows curves of the determinant of the system of equations (vibration amplitude) versus Froude number and vibration frequency for the dimensionless unity value of the rigidity R. The dark regions correspond to the minimum values of the determinant, i.e., to the resonant modes of motion. In Fig. 3, it is possible to distinguish two regions in which the Froude number and the vibration frequency take critical values, because of which the system enters a resonance. In addition, at zero vibration frequency, in Fig. 3 there is a peak corresponding to a sharp increase in the bending in the stationary mode of motion. This bending mode is shown in Fig. 4a. Figure 4c and d gives the resonant vibration modes of a hull made of an elastic material for the modes of motion corresponding to the extreme regions in Fig. 3. The vibration modes presented in Fig. 4 show the running behavior (the surface of the shell never coincides with the position of equilibrium). The two resonant regions shown in Fig. 3 end at Fr = 0 on the abscissa. In this case, the running property disappears and the resonant modes of shell vibrations are separated into symmetric and asymmetric modes (see Fig. 4b).

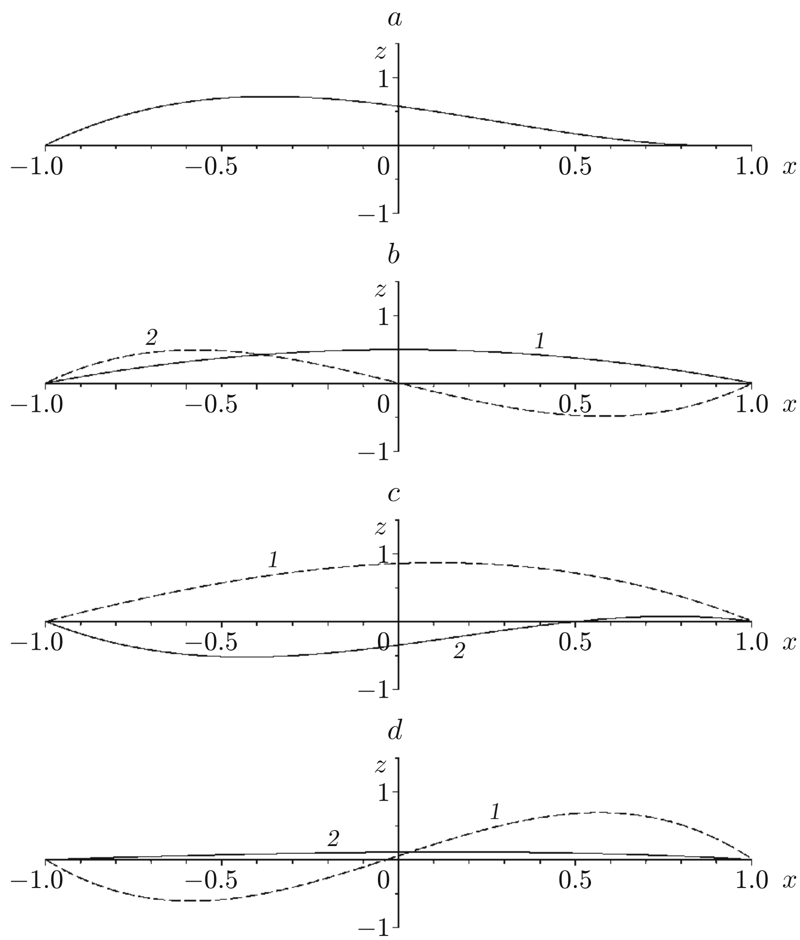

Fig. 4. Shell vibration modes in the diametral plane in various modes of motion: (a) $\mathrm{Fr} = 6.6$ and $\mathrm{Fr}\,k = 0$; (b) $\mathrm{Fr} = 0$ (curves 1 and 2 refer to $\mathrm{Fr}\,k = 12.9$ and 23.6, respectively); (c) $\mathrm{Fr} = 1.6$ (curve 1 refers to $\mathrm{Fr}\,k = 12.6$ and $t = T$; curve 2 refers to $\mathrm{Fr}\,k = 12.6$ and $t = T/2$); d) $\mathrm{Fr} = 3.1$ (curve 1 refers to $\mathrm{Fr}\,k = 24.8$ and $t = T$; curve 2 refers to $\mathrm{Fr}\,k = 24.8$ and $t = T/2$).

Conclusions. In the problem of nonstationary motion of a shell in a heavy incompressible fluid flow, formulas for the free-surface wave shapes and the natural frequencies of the shell were obtained using linear theory. The modes of motion accompanied by increased vibrations of the hull were found for an elastic shell of relative width $\lambda = 1/3$. The vibration amplitude was shown to depend on the Froude and Strouhal numbers. Fluid surface waves for various values of the Strouhal number were calculated. The calculation results can be useful in optimizing the hydrodynamic characteristics of high-velocity ships of new types (for example, Raketa-2) with hulls made of elastic materials.

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